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| 1 | $\overrightarrow{AB} = 2\mathbf{b} - 2\mathbf{a}$ or $\overrightarrow{BA} = 2\mathbf{a} - 2\mathbf{b}$ $\overrightarrow{MN} = 10\mathbf{a} - \mathbf{b}$ or $\overrightarrow{NM} = -10\mathbf{a} + \mathbf{b}$ | | 5 | M1 for finding \overrightarrow{AB} or \overrightarrow{BA} or \overrightarrow{MN} or \overrightarrow{NM} |
| | eg $\overrightarrow{MP} = -\mathbf{b} + 2\mathbf{a} + k(2\mathbf{b} - 2\mathbf{a})$ and $\overrightarrow{MP} = l(10\mathbf{a} - \mathbf{b})$ or eg $\overrightarrow{MP} = \mathbf{b} + k(2\mathbf{a} - 2\mathbf{b})$ and $\overrightarrow{MP} = l(10\mathbf{a} - \mathbf{b})$ or eg $\overrightarrow{PN} = 8\mathbf{a} + k(2\mathbf{a} - 2\mathbf{b})$ and $\overrightarrow{PN} = l(10\mathbf{a} - \mathbf{b})$ or eg $\overrightarrow{AP} = 8\mathbf{a} + k(\mathbf{b} - 10\mathbf{a})$ and $\overrightarrow{AP} = l(2\mathbf{b} - 2\mathbf{a})$ or eg $\overrightarrow{AP} = -2\mathbf{a} + \mathbf{b} + k(10\mathbf{a} - \mathbf{b})$ and $\overrightarrow{AP} = l(2\mathbf{b} - 2\mathbf{a})$ or eg $\overrightarrow{AM} = k(2\mathbf{b} - 2\mathbf{a}) + l(\mathbf{b} - 10\mathbf{a})$ and $\overrightarrow{AM} = -2\mathbf{a} + \mathbf{b}$ | | | M2 for writing eg \overrightarrow{MP} or \overrightarrow{PN} or \overrightarrow{AP} or \overrightarrow{AM} in two different ways in terms of \mathbf{a} and \mathbf{b} (M1 for writing eg \overrightarrow{MP} or \overrightarrow{PN} or \overrightarrow{AP} or \overrightarrow{AM} in one way) These may be written as eg \overrightarrow{PM} in place of \overrightarrow{MP} |
| | eg $2 - 2k = 10\lambda$ and $-1 + k = -\lambda$ (from \overrightarrow{MP} 1st) or eg $2k = 10\lambda$ and $1 - 2k = -\lambda$ (from \overrightarrow{MP} 2nd) or eg $8 + 2k = 10\lambda$ and $-2k = -\lambda$ (from \overrightarrow{PN}) or eg $8 - 10k = -2\lambda$ and $k = 2\lambda$ (from \overrightarrow{AP} 1st) or eg $-2 + 10k = -2\lambda$ and $1 - k = 2\lambda$ (from \overrightarrow{AP} 2nd) or eg $-2k - 10\lambda = -2$ and $2k + \lambda = 1$ (from \overrightarrow{AM}) | | | M1 dep M3 for writing a pair of equations using their variables. \overrightarrow{MP} (1st) leads to $\lambda = \frac{1}{9}, k = \frac{4}{9}$ \overrightarrow{MP} (2nd) leads to $\lambda = \frac{1}{9}, k = \frac{5}{9}$ \overrightarrow{PN} leads to $\lambda = \frac{8}{9}, k = \frac{4}{9}$ \overrightarrow{AP} (1st) leads to $\lambda = \frac{4}{9}, k = \frac{8}{9}$ \overrightarrow{AP} (2nd) leads to $\lambda = \frac{4}{9}, k = \frac{1}{9}$ \overrightarrow{AM} leads to $\lambda = \frac{1}{9}, k = \frac{4}{9}$ |
| | | 4 : 5 | | A1 cao |
| Total 5 marks | | | | |

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| 2 | $\overrightarrow{OC} = 3\mathbf{a} + 4\mathbf{b}$ $\overrightarrow{ON} = t(3\mathbf{a} + 4\mathbf{b})$ $\overrightarrow{ON} = 3\mathbf{a} + s(-3\mathbf{a} + 6\mathbf{b})$ $t(3\mathbf{a} + 4\mathbf{b}) = 3\mathbf{a} + s(-3\mathbf{a} + 6\mathbf{b})$ $\rightarrow t = 0.6, s = 0.4$ | | 5 | B1 Correct expression for \overrightarrow{OC} M1 Correct expressions for \overrightarrow{ON} M1 |
| | | $\overrightarrow{ON} = 1.8\mathbf{a} + 2.4\mathbf{b}$ oe | | A1 t or s value correct A1 e.g. $\overrightarrow{ON} = \frac{3}{5}(3\mathbf{a} + 4\mathbf{b})$ |
| | Alt: | | | |
| | $\overrightarrow{AB} = -3\mathbf{a} + 6\mathbf{b}$ $\overrightarrow{AN} = s(-3\mathbf{a} + 6\mathbf{b})$ $\overrightarrow{AN} = -3\mathbf{a} + t(3\mathbf{a} + 4\mathbf{b})$ $-3\mathbf{a} + t(3\mathbf{a} + 4\mathbf{b}) = s(-3\mathbf{a} + 6\mathbf{b})$ $\rightarrow t = 0.6, s = 0.4 \rightarrow \overrightarrow{AN} = -1.2\mathbf{a} + 2.4\mathbf{b}$ $\overrightarrow{ON} = 3\mathbf{a} + \overrightarrow{AN}$ | $\overrightarrow{ON} = 1.8\mathbf{a} + 2.4\mathbf{b}$ oe | | B1 Correct expression for \overrightarrow{AB} M1 Correct expressions for \overrightarrow{AN} M1 |
| | | | | A1 t or s value correct A1 e.g. $\overrightarrow{ON} = \frac{3}{5}(3\mathbf{a} + 4\mathbf{b})$ |
| | Alt: | | | |
| | $\overrightarrow{OC} = 3\mathbf{a} + 4\mathbf{b}$ $\overrightarrow{ON} : \overrightarrow{NC} = 6 : 4$ (ie 3:2) $\overrightarrow{ON} = \frac{3}{5}\overrightarrow{OC}$ | $\overrightarrow{ON} = 1.8\mathbf{a} + 2.4\mathbf{b}$ oe | | B1 Correct expression for \overrightarrow{OC} M1 M2 |
| | | | | A1 e.g. $\overrightarrow{ON} = \frac{3}{5}(3\mathbf{a} + 4\mathbf{b})$ |
| Total 5 marks | | | | |

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| 3 | $\overrightarrow{PM} = -\frac{3}{2}\mathbf{a} - \frac{3}{4}\mathbf{b} + 4\mathbf{a} + \frac{1}{2}(2\mathbf{b} - 4\mathbf{a}) \left(= \frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b} \right)$ $\overrightarrow{AM} = 4\mathbf{a} + \frac{1}{2}(2\mathbf{b} - 4\mathbf{a}) (= 2\mathbf{a} + \mathbf{b})$ $\overrightarrow{AM} = 2\mathbf{b} + \frac{1}{2}(4\mathbf{a} - 2\mathbf{b}) (= 2\mathbf{a} + \mathbf{b})$ $\overrightarrow{MA} = \frac{1}{2}(2\mathbf{b} - 4\mathbf{a}) - 2\mathbf{b} (= -2\mathbf{a} - \mathbf{b})$ $\overrightarrow{MA} = \frac{1}{2}(4\mathbf{a} - 2\mathbf{b}) - 4\mathbf{a} (= -2\mathbf{a} - \mathbf{b})$ | | 3 | M1 for finding \overrightarrow{PM} or \overrightarrow{AM} or \overrightarrow{MA} |
| | $(\overrightarrow{AP} : \overrightarrow{PM}) = \left \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b} \right : \left \frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b} \right \text{ oe}$ $(\overrightarrow{AP} : \overrightarrow{AM}) = \left \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b} \right : 2\mathbf{a} + \mathbf{b} (= 3 : 4) \text{ oe}$ $(\overrightarrow{AM} : \overrightarrow{PM}) = 2\mathbf{a} + \mathbf{b} : \left \frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b} \right (= 4 : 1) \text{ oe}$ $\overrightarrow{AP} = 3\overrightarrow{PM} \text{ oe eg } \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b} = 3\left(\frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b}\right) \text{ oe}$ $\overrightarrow{AM} = \frac{4}{3}\overrightarrow{AP} \text{ oe}$ $\overrightarrow{AM} = 4\overrightarrow{PM} \text{ oe}$ | | | M1 For use of a correct ratio or fraction linking \overrightarrow{AP} and \overrightarrow{PM} or \overrightarrow{AP} and \overrightarrow{AM} or \overrightarrow{AM} and \overrightarrow{PM} (in either order) vectors must be in form $p\mathbf{a} + q\mathbf{b}$ |
| | | 3 : 1 | | A1 |
| Total 3 marks | | | | |

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| 4 | $\text{eg } \overrightarrow{OP} = n(2\mathbf{a} + 3\mathbf{b})$ $\text{or } \overrightarrow{OP} = 2\mathbf{a} + m(5\mathbf{b} - 2\mathbf{a})$ $\text{or } \overrightarrow{OP} = 5\mathbf{b} + x(2\mathbf{a} - 5\mathbf{b})$ | | 5 | M1 for a vector equation for \overrightarrow{OP} |
| | $\text{eg } \overrightarrow{OP} = n(2\mathbf{a} + 3\mathbf{b}) \text{ and } \overrightarrow{OP} = 2\mathbf{a} + m(5\mathbf{b} - 2\mathbf{a})$ or $\text{eg } \overrightarrow{OP} = n(2\mathbf{a} + 3\mathbf{b}) \text{ and } \overrightarrow{OP} = 5\mathbf{b} + x(2\mathbf{a} - 5\mathbf{b}) \text{ oe}$ | | | M1 2 vector equations for \overrightarrow{OP} that can be used to find \overrightarrow{OP} - must be in terms of \mathbf{a} and \mathbf{b} and a scalar |
| | $\text{eg } 5m = 3n \text{ or } m = \frac{3}{5}n \text{ or } 2n = 2 - 2m \text{ or } n = 1 - m \text{ oe}$ $\text{and } 2 - 2 \times \frac{3}{5}n = 2n \text{ or } 2 \times \frac{5}{3}m = 2 - 2m \text{ oe}$ or $\text{eg } 2n = 2x \text{ or } n = x \text{ or } 3n = 5 - 5x \text{ oe}$ $\text{and } 3x = 5 - 5x \text{ or } 3n = 5 - 5n \text{ oe}$ | | | M1 Writing one equation in terms of only one scalar eg one of n or m or x etc |
| | $\text{eg } m = \frac{3}{8} \text{ or } n = \frac{5}{8} \text{ or } x = \frac{5}{8} \text{ oe}$ | | | M1 for a correct value for one scalar |
| | Working is required | $\frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}$ | | A1 oe (dep on M1) but terms in \mathbf{a} and terms in \mathbf{b} should be simplified. eg $\frac{1}{8}(10\mathbf{a} + 15\mathbf{b})$ or $\frac{5}{8}(2\mathbf{a} + 3\mathbf{b})$ etc |
| Total 5 marks | | | | |
| Alternative method as a vector method not requested | | | | |
| | $\text{eg } \overrightarrow{OP} = n(2\mathbf{a} + 3\mathbf{b})$ | | 5 | M1 for a vector equation for \overrightarrow{OP} |

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| | $\text{eg } \overrightarrow{CP} : \overrightarrow{OP} = 3 : 5 \text{ or } \overrightarrow{CP} : \overrightarrow{CO} = 3 : 8 \text{ or}$ $\frac{\overrightarrow{CP}}{\overrightarrow{OP}} = \frac{3}{5} \text{ or } \frac{\overrightarrow{CP}}{\overrightarrow{CO}} = \frac{3}{8} \text{ oe}$ | | | M2 for a correct ratio for two sides in triangle ACP and triangle BOP that help to find \overrightarrow{OP} as a fraction of \overrightarrow{OC} (could be seen on the diagram) |
| | $\overrightarrow{OP} = \frac{5}{8}\overrightarrow{OC} \text{ or } n = \frac{5}{8}$ | | | M1 |
| | Working is required | $\frac{5}{4}\mathbf{a} + \frac{15}{8}\mathbf{b}$ | | A1 oe (dep on M1) but terms in \mathbf{a} and terms in \mathbf{b} should be simplified. eg $\frac{1}{8}(10\mathbf{a} + 15\mathbf{b})$ or $\frac{5}{8}(2\mathbf{a} + 3\mathbf{b})$ etc |
| Total 5 marks | | | | |

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| 5 | $\text{e.g. } \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \end{pmatrix} \text{ or } \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ | | 2 | M1 or for $\begin{pmatrix} 7 \\ a \end{pmatrix}$ where $a \neq -1$ or $\begin{pmatrix} b \\ -1 \end{pmatrix}$ where $b \neq 7$ |
| | | $\begin{pmatrix} 7 \\ -1 \end{pmatrix}$ | | A1 |
| Total 2 marks | | | | |

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| 6 | $\overrightarrow{AB} = 2\mathbf{b} - 2\mathbf{a}$ oe or $\overrightarrow{BA} = 2\mathbf{a} - 2\mathbf{b}$ oe or $\overrightarrow{AM} = \mathbf{b} - \mathbf{a}$ oe or $\overrightarrow{MA} = \mathbf{a} - \mathbf{b}$ oe or $\overrightarrow{BM} = \mathbf{b} - \mathbf{a}$ oe or $\overrightarrow{MB} = \mathbf{a} - \mathbf{b}$ oe | | 6 | M1 for finding \overrightarrow{AB} or \overrightarrow{BA} or \overrightarrow{AM} or \overrightarrow{MA} or \overrightarrow{BM} or \overrightarrow{MB} |
| | e.g. $\overrightarrow{OM} = 2\mathbf{a} + (\mathbf{b} - \mathbf{a}) (= \mathbf{a} + \mathbf{b})$ oe or $\overrightarrow{MO} = (\mathbf{b} - \mathbf{a}) - 2\mathbf{b} (= -\mathbf{a} - \mathbf{b})$ oe or $\overrightarrow{AN} = \frac{4}{3}\mathbf{b} - 2\mathbf{a}$ oe or $\overrightarrow{NA} = 2\mathbf{a} - \frac{4}{3}\mathbf{b}$ oe | | | M1 for finding \overrightarrow{OM} or \overrightarrow{MO} or \overrightarrow{AN} or \overrightarrow{NA} |
| | e.g. $\overrightarrow{OP} = 2\mathbf{a} + \lambda\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right)$ oe or $\overrightarrow{OP} = \frac{4}{3}\mathbf{b} + \lambda\left(2\mathbf{a} - \frac{4}{3}\mathbf{b}\right)$ oe or $\overrightarrow{OP} = \mu(\mathbf{a} + \mathbf{b})$ oe OR $\overrightarrow{MP} = \mathbf{a} - \mathbf{b} + k\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right)$ oe or $\overrightarrow{MP} = \mathbf{b} - \mathbf{a} - \frac{2}{3}\mathbf{b} + k\left(2\mathbf{a} - \frac{4}{3}\mathbf{b}\right)$ oe or $\overrightarrow{MP} = t(-\mathbf{a} - \mathbf{b})$ oe | | | M1 for finding \overrightarrow{OP} or \overrightarrow{PO} or \overrightarrow{MP} or \overrightarrow{PM} |
| | e.g. $2\mathbf{a} + \lambda\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right) = \mu(\mathbf{a} + \mathbf{b})$ oe or $\frac{4}{3}\mathbf{b} + \lambda\left(2\mathbf{a} - \frac{4}{3}\mathbf{b}\right) = \mu(\mathbf{a} + \mathbf{b})$ oe or $\mathbf{a} - \mathbf{b} + k\left(\frac{4}{3}\mathbf{b} - 2\mathbf{a}\right) = t(-\mathbf{a} - \mathbf{b})$ oe or $\mathbf{b} - \mathbf{a} - \frac{2}{3}\mathbf{b} + k\left(2\mathbf{a} - \frac{4}{3}\mathbf{b}\right) = t(-\mathbf{a} - \mathbf{b})$ oe | | | M1 for setting up an equation for \overrightarrow{OP} or \overrightarrow{MP} |
| | $\mu = \frac{4}{5}$ or $t = \frac{1}{5}$ | | | M1 for finding μ or t for either $\overrightarrow{OP} = \mu\overrightarrow{OM}$ or $\overrightarrow{MP} = t\overrightarrow{MO}$ |
| | | 4 : 1 | | A1 cao (dep on M3) |
| Total 6 marks | | | | |

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| 7 | eg $\overrightarrow{AK} = \lambda\mathbf{a}$ $\overrightarrow{KB} = (1 - \lambda)\mathbf{a}$ $\overrightarrow{CL} = -\mu\mathbf{a}$ $\overrightarrow{DL} = (1 - \mu)\mathbf{a}$ | eg $\overrightarrow{AK} = \frac{1}{2}\mu\mathbf{a}$ $\overrightarrow{KB} = (1 - \frac{1}{2}\mu)\mathbf{a}$ $\overrightarrow{CL} = -2\lambda\mathbf{a}$ $\overrightarrow{DL} = (1 - 2\lambda)\mathbf{a}$ | SEE NEXT PAGE FOR MISREAD | 5 | M1 for correctly using the ratio to form an expression for a vector passing through K or L could be in terms of λ or μ \overrightarrow{AK} or \overrightarrow{KA} , \overrightarrow{KB} or \overrightarrow{BK} , \overrightarrow{CL} or \overrightarrow{LC} , \overrightarrow{DL} or \overrightarrow{LD} (may be seen as part of another expression) |
| | eg $\overrightarrow{KL} = -\lambda\mathbf{a} + \mathbf{b} + (1 - \mu)\mathbf{a}$ or $= (1 - \lambda - \mu)\mathbf{a} + \mathbf{b}$ $\overrightarrow{LM} = (\mu - 1)\mathbf{a} + 0.5\mathbf{b}$ $\overrightarrow{KM} = -\lambda\mathbf{a} + \mathbf{b} + 0.5\mathbf{b} (= -\lambda\mathbf{a} + 1.5\mathbf{b})$ | eg $\overrightarrow{KL} = \mathbf{b} + (1 - \frac{3}{2}\mu)\mathbf{a}$ or $\overrightarrow{KL} = \mathbf{b} + (1 - 3\lambda)\mathbf{a}$ $\overrightarrow{LM} = (2\lambda - 1)\mathbf{a} + \frac{1}{2}\mathbf{b}$ or $\overrightarrow{KM} = -\lambda\mathbf{a} + \frac{3}{2}\mathbf{b}$ or | | | M1 for finding an expression in λ and/or μ for one of \overrightarrow{KL} (or \overrightarrow{LK}), \overrightarrow{LM} (or \overrightarrow{ML}), \overrightarrow{KM} (or \overrightarrow{MK}) [If this mark is awarded it assumes the first M1] |
| | Two of the above – may have used $2\lambda = \mu$ to write all in one of λ or μ May be simplified or not – so may have brackets or not | | | | M1 for finding an expression in λ or μ for two of the following: \overrightarrow{KL} (or \overrightarrow{LK}), \overrightarrow{LM} (or \overrightarrow{ML}), or \overrightarrow{KM} (or \overrightarrow{MK}) |
| | eg using $\overrightarrow{KM} = -\lambda\mathbf{a} + 1.5\mathbf{b}$ and $\overrightarrow{LM} = (2\lambda - 1)\mathbf{a} + \frac{1}{2}\mathbf{b}$ $\overrightarrow{LM} = x\overrightarrow{KM}$ gives $\frac{-\lambda x}{2\lambda - 1} = \frac{1.5x}{0.5} \Rightarrow 3.5\lambda = 1.5 \Rightarrow \lambda = \frac{3}{7}$ oe | | $\lambda = \frac{3}{7}$ or $\mu = \frac{6}{7}$ | | A1 dep on M2 for one value correct or both values but written the wrong way round ($\mu = \frac{3}{7}$ $\lambda = \frac{6}{7}$) |
| | | | $\lambda = \frac{3}{7}$ & $\mu = \frac{6}{7}$ | | A1 dep on M2 for both values |
| Total 5 marks | | | | | |

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| 7 | $\text{eg } \overrightarrow{AK} = \left(\frac{\lambda}{\lambda+1} \right) \mathbf{a}$ $\overrightarrow{KB} = \left(\frac{1}{\lambda+1} \right) \mathbf{a}$ $\overrightarrow{CL} = \left(\frac{-\mu}{1+\mu} \right) \mathbf{a}$ $\overrightarrow{DL} = \left(\frac{1}{1+\mu} \right) \mathbf{a}$ | $\text{eg } \overrightarrow{AK} = \left(\frac{\frac{1}{2}\mu}{\frac{1}{2}\mu+1} \right) \mathbf{a} = \left(\frac{\mu}{\mu+2} \right) \mathbf{a}$ $\overrightarrow{KB} = \left(\frac{1}{\frac{1}{2}\mu+1} \right) \mathbf{a} = \left(\frac{2}{\mu+2} \right) \mathbf{a}$ $\overrightarrow{CL} = \left(\frac{-2\lambda}{1+2\lambda} \right) \mathbf{a}$ $\overrightarrow{DL} = \left(\frac{1}{1+2\lambda} \right) \mathbf{a}$ | <p>MISREAD $\overrightarrow{AK}:\overrightarrow{KB}=\lambda:1$ and $\overrightarrow{CL}:\overrightarrow{LD}=\mu:1$</p> | <p>M1 For using the ratio to form an expression for a vector passing through K or L could be in terms of λ or μ \overrightarrow{AK} or \overrightarrow{KA}, \overrightarrow{KB} or \overrightarrow{BK}, \overrightarrow{CL} or \overrightarrow{LC}, \overrightarrow{DL} or \overrightarrow{LD} (may be seen as part of another expression)</p> |
| | $\text{eg } \overrightarrow{KL} = \left(\frac{-\lambda}{\lambda+1} \right) \mathbf{a} + \mathbf{b} + \left(\frac{1}{1+\mu} \right) \mathbf{a} \text{ or}$ $\overrightarrow{LM} = \left(\frac{-1}{1+\mu} \right) \mathbf{a} + 0.5\mathbf{b}$ $\overrightarrow{KM} = \left(\frac{-\lambda}{1+\lambda} \right) \mathbf{a} + \frac{3}{2}\mathbf{b} \text{ oe}$ | $\text{eg } \overrightarrow{KL} = \left(\frac{-\frac{1}{2}\mu}{\frac{1}{2}\mu+1} \right) \mathbf{a} + \mathbf{b} + \left(\frac{1}{1+\mu} \right) \mathbf{a}$ $\overrightarrow{LM} = \left(\frac{-1}{1+2\lambda} \right) \mathbf{a} + \frac{1}{2}\mathbf{b} \text{ or}$ $\overrightarrow{KM} = \left(\frac{-\frac{1}{2}\mu}{\frac{1}{2}\mu+1} \right) \mathbf{a} + \frac{3}{2}\mathbf{b} \text{ oe}$ | | <p>M1 for finding an expression in λ and/or μ using the above misread for one of \overrightarrow{KL} (or \overrightarrow{LK}), \overrightarrow{LM} (or \overrightarrow{ML}), \overrightarrow{KM} (or \overrightarrow{MK}) [If this mark is awarded it assumes the first M1]</p> |
| | Two of the above – may have used $2\lambda = \mu$ to write all in one of λ or μ May be simplified or not – so may have brackets or not | | | <p>M1 for finding an expression in λ or μ for two of \overrightarrow{KL} (or \overrightarrow{LK}), \overrightarrow{LM} (or \overrightarrow{ML}), \overrightarrow{KM} (or \overrightarrow{MK})</p> |
| | (Giving answers of $\lambda = 0.5(1+\sqrt{7})$, $\mu = 1+\sqrt{7}$) | | | <p>A MAXIMUM OF 3 MARKS CAN BE AWARDED FOR THIS MISREAD</p> |

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| 8 | $\overrightarrow{OP} = 4\mathbf{a} + 2\mathbf{a} + 8\mathbf{b} (= 6\mathbf{a} + 8\mathbf{b}) \text{ oe OR } \overrightarrow{PO} = -6\mathbf{a} - 8\mathbf{b} \text{ oe or}$ $\overrightarrow{AB} = 6\mathbf{b} - 4\mathbf{a} \text{ oe OR } \overrightarrow{BA} = 4\mathbf{a} - 6\mathbf{b} \text{ oe or}$ $\overrightarrow{BP} = 6\mathbf{a} + 2\mathbf{b} \text{ oe OR } \overrightarrow{PB} = -6\mathbf{a} - 2\mathbf{b} \text{ oe}$ | | 5 | <p>M1 oe for one of \overrightarrow{OP} or \overrightarrow{PO} or \overrightarrow{AB} or \overrightarrow{BA} or \overrightarrow{BP} or \overrightarrow{PB} (may be seen as part of another vector calculation)</p> |
| | or $\overrightarrow{OQ} = 4\mathbf{a} + \lambda(6\mathbf{b} - 4\mathbf{a}) \text{ oe OR } 6\mathbf{b} + \mu(4\mathbf{a} - 6\mathbf{b}) \text{ oe OR } x(6\mathbf{a} + 8\mathbf{b}) \text{ oe}$ or $\overrightarrow{BQ} = \mu(4\mathbf{a} - 6\mathbf{b}) \text{ oe OR } -6\mathbf{b} + \lambda(6\mathbf{a} + 8\mathbf{b}) \text{ oe OR } 4\mathbf{a} - 6\mathbf{b} + x(6\mathbf{b} - 4\mathbf{a}) \text{ oe}$ or $\overrightarrow{AQ} = y(6\mathbf{b} - 4\mathbf{a}) \text{ oe OR } -4\mathbf{a} + x(6\mathbf{a} + 8\mathbf{b}) \text{ oe OR } 6\mathbf{b} - 4\mathbf{a} + \mu(4\mathbf{a} - 6\mathbf{b}) \text{ oe OR}$ $2\mathbf{a} + 8\mathbf{b} + m(6\mathbf{a} + 8\mathbf{b}) \text{ oe}$ or $\overrightarrow{QP} = \lambda(6\mathbf{a} + 8\mathbf{b}) \text{ oe OR } \mu(4\mathbf{a} - 6\mathbf{b}) + 2\mathbf{a} + 8\mathbf{b} \text{ oe}$ | | | <p>M1 for one of \overrightarrow{OQ} or \overrightarrow{QO} or \overrightarrow{BQ} or \overrightarrow{QB} or \overrightarrow{AQ} or \overrightarrow{QA} or \overrightarrow{QP} or \overrightarrow{PQ}</p> |
| | | | | <p>M1 for a second correct expression for the same vector OR for two correct expressions for parallel vectors eg 2 of \overrightarrow{OQ}, \overrightarrow{OP}, \overrightarrow{QP} oe AND using ratios to form an equation in one variable that can lead to a solution eg $\overrightarrow{OQ} = 4\mathbf{a} + k(6\mathbf{b} - 4\mathbf{a})$ and $\overrightarrow{QP} = 2\mathbf{a} + 8\mathbf{b} - k(6\mathbf{b} - 4\mathbf{a})$ and $\frac{4-4k}{2+4k} = \frac{6k}{8-6k}$</p> |
| | $\text{eg } \lambda = \frac{8}{17} \text{ or } \mu = \frac{9}{17} \text{ or } \overrightarrow{AQ}:\overrightarrow{QB} = \frac{4x}{3}:\frac{3x}{2} \text{ oe}$ | | | <p>A1 oe</p> |
| | | 8 : 9 | | <p>A1 oe</p> |

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| 9 | $(\overrightarrow{ON} = \lambda(\mathbf{a} + \mathbf{b})) (= \lambda\mathbf{a} + \lambda\mathbf{b}) \text{ or}$ $(\overrightarrow{NY} = (1-\lambda)(\mathbf{a} + \mathbf{b})) (= (1-\lambda)\mathbf{a} + (1-\lambda)\mathbf{b})$ | | 5 | <p>M1 for finding a vector for \overrightarrow{ON} or \overrightarrow{NY} or \overrightarrow{NO} or \overrightarrow{YN} in terms of \mathbf{a} and \mathbf{b} and using λ oe (can be embedded)</p> |
| | $(\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}) = -0.5\mathbf{a} + \lambda\mathbf{a} + \lambda\mathbf{b} (= (\lambda - 0.5)\mathbf{a} + \lambda\mathbf{b}) \text{ or } (\overrightarrow{MZ} = \overrightarrow{MO} + \overrightarrow{OZ}) = -0.5\mathbf{a} + 3\mathbf{b}$ $\text{or } (\overrightarrow{MN} = \overrightarrow{MX} + \overrightarrow{XY} + \overrightarrow{YN}) = 0.5\mathbf{a} + \mathbf{b} + (\lambda - 1)(\mathbf{a} + \mathbf{b}) (= (\lambda - 0.5)\mathbf{a} + \lambda\mathbf{b})$ | | | <p>M1 for finding a vector for \overrightarrow{MN} or \overrightarrow{NM} or \overrightarrow{MZ} or \overrightarrow{ZM}</p> |
| | $(\overrightarrow{MN} = \mu\overrightarrow{MZ}) = \mu(-0.5\mathbf{a} + 3\mathbf{b}) (= -0.5\mu\mathbf{a} + 3\mu\mathbf{b}) \text{ or}$ $(\overrightarrow{ON} = \overrightarrow{OM} + \overrightarrow{MN}) = 0.5\mathbf{a} + \mu(-0.5\mathbf{a} + 3\mathbf{b}) (= (0.5 - 0.5\mu)\mathbf{a} + 3\mu\mathbf{b}) \text{ or}$ $(\overrightarrow{NY} = \overrightarrow{NM} + \overrightarrow{MX} + \overrightarrow{XY}) = -\mu(-0.5\mathbf{a} + 3\mathbf{b}) + 0.5\mathbf{a} + \mathbf{b} (= (0.5 + 0.5\mu)\mathbf{a} + (1 - 3\mu)\mathbf{b})$ | | | <p>M1 for finding a vector for \overrightarrow{MN} or \overrightarrow{ON} or \overrightarrow{NY} or \overrightarrow{NM} or \overrightarrow{NO} or \overrightarrow{YN} using another variable e.g. μ oe</p> |
| | $-0.5\mu = -0.5 + \lambda \text{ oe}$ $3\mu = \lambda \text{ oe}$ | $1 - \lambda = 0.5\mu + 0.5 \text{ oe}$ $1 - \lambda = 1 - 3\mu \text{ oe}$ | | <p>M1 for setting up two simultaneous equations using the components of \mathbf{a} and \mathbf{b} for \overrightarrow{MN} or \overrightarrow{ON} or \overrightarrow{NY} oe</p> |
| | | | $\frac{3}{7}$ | <p>A1 (allow $\frac{3}{7} = 0.42(8571\dots)$ to 2 sf truncated or rounded)</p> |
| | | | | <p>Total 5 marks</p> |

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| 9 ALT | $(\overrightarrow{ON} = \lambda(\mathbf{a} + \mathbf{b}) = \lambda\mathbf{a} + \lambda\mathbf{b})$ or $(\overrightarrow{NY} = (1 - \lambda)(\mathbf{a} + \mathbf{b}) = (1 - \lambda)\mathbf{a} + (1 - \lambda)\mathbf{b})$ | | 5 | M1 for finding a vector for \overrightarrow{ON} or \overrightarrow{NY} or \overrightarrow{NO} or \overrightarrow{YN} in terms a and b and using λ oe |
| | $(\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON} = -0.5\mathbf{a} + \lambda\mathbf{a} + \lambda\mathbf{b} = (\lambda - 0.5)\mathbf{a} + \lambda\mathbf{b})$ or $(\overrightarrow{MN} = \overrightarrow{MX} + \overrightarrow{XY} + \overrightarrow{YN} = 0.5\mathbf{a} + \mathbf{b} + (\lambda - 1)(\mathbf{a} + \mathbf{b}) = (\lambda - 0.5)\mathbf{a} + \lambda\mathbf{b})$ | | | M1 for finding a vector for \overrightarrow{MN} or \overrightarrow{NM} in terms a and b and using λ oe |
| | $(\overrightarrow{NZ} = \overrightarrow{NO} + \overrightarrow{OZ} = -\lambda(\mathbf{a} + \mathbf{b}) + 3\mathbf{b} = -\lambda\mathbf{a} + (3 - \lambda)\mathbf{b})$ or $(\overrightarrow{NZ} = \overrightarrow{NY} + \overrightarrow{YZ} = (1 - \lambda)(\mathbf{a} + \mathbf{b}) - \mathbf{b} - \mathbf{a} + 3\mathbf{b} = -\lambda\mathbf{a} + (3 - \lambda)\mathbf{b})$ | | | M1 for finding a vector for \overrightarrow{NZ} or \overrightarrow{ZN} in terms a and b and using λ oe |
| | $\frac{\lambda - 0.5}{-\lambda} = \frac{\lambda}{3 - \lambda}$ oe | | | M1 for setting up an equation using the components of \overrightarrow{MN} and \overrightarrow{NZ} oe |
| | | $\frac{3}{7}$ | | A1 (allow $\frac{3}{7} = 0.42(8571\dots)$ to 2 sf truncated or rounded) |
| Total 5 marks | | | | |

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| 10 | eg $\overrightarrow{ON} = 8\mathbf{a} + \frac{1}{2}(6\mathbf{b} - 8\mathbf{a}) = 3\mathbf{b} + 4\mathbf{a}$ or $\overrightarrow{ON} = 6\mathbf{b} + \frac{1}{2}(-6\mathbf{b} + 8\mathbf{a}) = 3\mathbf{b} + 4\mathbf{a}$ or $\overrightarrow{NO} = \frac{1}{2}(8\mathbf{a} - 6\mathbf{b}) - 8\mathbf{a} = -4\mathbf{a} - 3\mathbf{b}$ or $\overrightarrow{NO} = -6\mathbf{b} + \frac{1}{2}(6\mathbf{b} - 8\mathbf{a}) = -3\mathbf{b} - 4\mathbf{a}$ or $\overrightarrow{AM} = -8\mathbf{a} + \frac{1}{3}(6\mathbf{b}) = 2\mathbf{b} - 8\mathbf{a}$ or $\overrightarrow{AM} = -8\mathbf{a} + 6\mathbf{b} - \frac{2}{3}(6\mathbf{b}) = 2\mathbf{b} - 8\mathbf{a}$ or $\overrightarrow{MA} = 8\mathbf{a} - \frac{1}{3}(6\mathbf{b}) = 8\mathbf{a} - 2\mathbf{b}$ or $\overrightarrow{MA} = \frac{4}{3}(6\mathbf{b}) + 8\mathbf{a} - 6\mathbf{b} = 8\mathbf{a} - 2\mathbf{b}$ | | 5 | M1 a correct expression for \overrightarrow{ON} or \overrightarrow{NO} or \overrightarrow{AM} or \overrightarrow{MA} |
| | $\overrightarrow{OP} = \mu(3\mathbf{b} + 4\mathbf{a})$ and one of eg $\overrightarrow{OP} = 8\mathbf{a} + x(2\mathbf{b} - 8\mathbf{a}) = (8 - 8x)\mathbf{a} + 2x\mathbf{b}$ or $\overrightarrow{OP} = 2\mathbf{b} + y(8\mathbf{a} - 2\mathbf{b}) = (2 - 2y)\mathbf{b} + 8y\mathbf{a}$ | | | M2 oe (M1 for one correct expression for \overrightarrow{OP}) (where μ, x, y are scalars) |
| | eg $\frac{4}{3} = \frac{8y}{2 - 2y}$ or $\frac{4}{3} = \frac{8 - 8x}{2x}$ oe or $3\mu = 2x$ and $4\mu = 8 - 8x$ or $3\mu = 2 - 2y$ and $4\mu = 8y$ | | | M1 A correct expression to find the position of P along ON or two correct simultaneous equations coming from the expressions for \overrightarrow{OP} |
| | | $2\mathbf{a} + \frac{3}{2}\mathbf{b}$ | | A1 dep on M3, oe eg $2\mathbf{a} + 1.5\mathbf{b}$ |
| Total 5 marks | | | | |

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| 11 (a) | $\overrightarrow{ON} = \mathbf{b} + \frac{2}{5}(\mathbf{a} - \mathbf{b})$ oe or $\overrightarrow{ON} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$ oe | | 2 | M1 |
| | Correct answer scores full marks (unless from obvious incorrect working) | $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ | | A1 oe eg $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ but must be one term in a and one in b |

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| 12 | eg $-\begin{pmatrix} 5 \\ 4 \end{pmatrix} + \begin{pmatrix} 9 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 5 \\ -4 \end{pmatrix} + \begin{pmatrix} 9 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 14 \\ a \end{pmatrix} a \neq -3$ or $\begin{pmatrix} b \\ -3 \end{pmatrix} b \neq 14$ | | 2 | M1 or an answer of $\begin{pmatrix} -14 \\ 3 \end{pmatrix}$ |
| | Correct answer scores full marks (unless from obvious incorrect working) | $\begin{pmatrix} 14 \\ -3 \end{pmatrix}$ | | A1 |
| Total 2 marks | | | | |

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| 13 | e.g. $(\overrightarrow{AB}) = 2\mathbf{b} - 2\mathbf{a}$ oe or $(\overrightarrow{BA}) = 2\mathbf{a} - 2\mathbf{b}$ oe or $(\overrightarrow{BD}) = 2(2\mathbf{b} - 2\mathbf{a}) = 4\mathbf{b} - 4\mathbf{a}$ oe or $(\overrightarrow{AD}) = 3(2\mathbf{b} - 2\mathbf{a}) = 6\mathbf{b} - 6\mathbf{a}$ oe | | 5 | M1 |
| | e.g. $(\overrightarrow{OE}) = 2\mathbf{b} + 2(2\mathbf{b} - 2\mathbf{a}) + 7\mathbf{a} + 3\mathbf{b} = 3\mathbf{a} + 9\mathbf{b}$ oe or $(\overrightarrow{OC}) = 2\mathbf{a} + \lambda(2\mathbf{b} - 2\mathbf{a}) = (2 - 2\lambda)\mathbf{a} + 2\lambda\mathbf{b}$ oe or $2\mathbf{b} + \lambda(2\mathbf{a} - 2\mathbf{b})$ or $(\overrightarrow{CE}) = (2\mathbf{b} - 2\mathbf{a}) - \lambda(2\mathbf{b} - 2\mathbf{a}) + 2(2\mathbf{b} - 2\mathbf{a}) + 7\mathbf{a} + 3\mathbf{b} = (1 + 2\lambda)\mathbf{a} + (9 - 2\lambda)\mathbf{b}$ | | | M2 for 2 correct paths seen M1 for 1 correct path seen Any correct path for OC passing through A or B involving a variable |
| | e.g. $\frac{2 - 2\lambda}{2\lambda} = \frac{1 + 2\lambda}{9 - 2\lambda}$ oe or $\frac{2 - 2\lambda}{2\lambda} = \frac{3}{9}$ oe or $\frac{(1 + 2\lambda)}{(9 - 2\lambda)} = \frac{1}{3}$ oe or $\lambda = \frac{3}{4}$ or $(2 - 2\lambda)\mathbf{a} + 2\lambda\mathbf{b} = \mu(3\mathbf{a} + 9\mathbf{b})$ or $\lambda = \frac{3}{4}$ or $\mu = \frac{1}{6}$ or $2\mathbf{b} + \lambda(2\mathbf{a} - 2\mathbf{b}) = \mu(3\mathbf{a} + 9\mathbf{b})$ or $\lambda = \frac{1}{4}$ or $\mu = \frac{5}{6}$ | | | M1 for comparing coefficients of a and b for $(OC$ and $CE)$ or $(OC$ and $OE)$ or $(CE$ and $OE)$ OC is a multiple of OE Two different paths for OC |
| | | 1 : 5 | | A1 dep M2 oe e.g 2 : 10 |
| | Working required | | | Total 5 marks |

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| 13 | e.g. $(\overrightarrow{AB}) = 2\mathbf{b} - 2\mathbf{a}$ oe or $(\overrightarrow{BA}) = 2\mathbf{a} - 2\mathbf{b}$ oe or $(\overrightarrow{BD}) = 2(2\mathbf{b} - 2\mathbf{a}) (= 4\mathbf{b} - 4\mathbf{a})$ oe or $(\overrightarrow{AD}) = 3(2\mathbf{b} - 2\mathbf{a}) (= 6\mathbf{b} - 6\mathbf{a})$ oe | | 5 | M1 |
| | e.g. $(\overrightarrow{OE}) = 2\mathbf{b} + 2(2\mathbf{b} - 2\mathbf{a}) + 7\mathbf{a} + 3\mathbf{b} (= 3\mathbf{a} + 9\mathbf{b})$ oe | | | M1 |
| | e.g. $(\overrightarrow{AE}) = 2\mathbf{b} - 2\mathbf{a} + 2(2\mathbf{b} - 2\mathbf{a}) + 7\mathbf{a} + 3\mathbf{b} (= \mathbf{a} + 9\mathbf{b})$ oe | | | M1 |
| | $[\overrightarrow{AE} = \lambda \overrightarrow{AD} + \mu \overrightarrow{OE}]$ $\mathbf{a} + 9\mathbf{b} = \lambda(6\mathbf{b} - 6\mathbf{a}) + \mu(3\mathbf{a} + 9\mathbf{b})$ oe or $1 = -6\lambda + 3\mu$ oe and $9 = 6\lambda + 9\mu$ oe or or $\lambda = \frac{1}{4}$ or $\mu = \frac{5}{6}$ | | | M1 |
| | | 1 : 5 | | A1 dep on M2 oe e.g. 2 : 10 |
| | <i>Working required</i> | | | Total 5 marks |

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| 14 | (a) | | - 2a + b | 1 | B1 | oe |
| | (b) | eg $\overrightarrow{OP} = \mathbf{a} + y(-\mathbf{a} + 6\mathbf{b})$ and $\overrightarrow{OP} = 2\mathbf{a} + x(-2\mathbf{a} + \mathbf{b})$ $\overrightarrow{AP} = -\mathbf{a} + y(-\mathbf{a} + 6\mathbf{b})$ and $\overrightarrow{AP} = x(-2\mathbf{a} + \mathbf{b})$ $\overrightarrow{AB} = x(-2\mathbf{a} + \mathbf{b}) + y(-\mathbf{a} + 6\mathbf{b})$ and $\overrightarrow{AB} = -2\mathbf{a} + 6\mathbf{b}$ $\overrightarrow{NP} = -\mathbf{b} + \mathbf{a} + y(-\mathbf{a} + 6\mathbf{b})$ and $\overrightarrow{NP} = x(2\mathbf{a} - \mathbf{b})$ $\overrightarrow{MP} = x(-\mathbf{a} + 6\mathbf{b})$ and $\overrightarrow{MP} = \mathbf{a} + y(-2\mathbf{a} + \mathbf{b})$ | | 4 | M2 | ft from (a), for writing eg \overrightarrow{OP} or \overrightarrow{AP} or \overrightarrow{AB} or \overrightarrow{NP} or \overrightarrow{MP} or similar in two different ways in terms of a and b (M1 for writing eg \overrightarrow{OP} or \overrightarrow{AP} or \overrightarrow{AB} or \overrightarrow{NP} or \overrightarrow{MP} or similar in one way in terms of a and b) These may be written as eg \overrightarrow{PO} in place of \overrightarrow{OP} |
| | | eg $x = 6y$ and $2 - 2x = 1 - y$ (from \overrightarrow{OP}) $x = 6y$ and $-2x = -1 - y$ (from \overrightarrow{AP}) $6 = x + 6y$ and $-2 = -2x - y$ (from \overrightarrow{AB}) $2x = 1 - y$ and $-x = -1 + 6y$ (from \overrightarrow{NP}) $-x = 1 - 2y$ and $6x = y$ (from \overrightarrow{MP}) | | | M1 | dep M2 for writing a pair of equations using their variables \overrightarrow{OP} leads to $x = \frac{6}{11}, y = \frac{1}{11}$ \overrightarrow{AP} leads to $x = \frac{6}{11}, y = \frac{1}{11}$ \overrightarrow{AB} leads to $x = \frac{6}{11}, y = \frac{10}{11}$ \overrightarrow{NP} leads to $x = \frac{5}{11}, y = \frac{1}{11}$ \overrightarrow{MP} leads to $x = \frac{1}{11}, y = \frac{6}{11}$ |
| | | Vector method required | 6 : 5 | | A1 | dep on M2, oe eg $\frac{6}{11} : \frac{5}{11}$ |
| | | | | | | Total 5 marks |